Sample Aptitude Test Mark Scheme

General Advice

- Give yourself plenty of space
- Write one thing on each line
- Make your diagrams large enough
- Write fractions like this $\frac{3}{4}$ not ¾
- If in doubt, put more working not less
- Check your answers where possible, for example in an algebra question sub your answer back in to make sure it works.
- If you are stuck, then try to think what bit of maths you might be able to use (e.g. Pythagoras' Theorem in a right-angled triangle) even if it doesn't lead directly to an answer, it may be halfway there.

1. Sami buys 25 mugs for £0.84 each. He gives away 4 of them and sells the rest at £1.40 each.

What percentage profit does he make?

Solution

It's a good idea to write in words what each step of your working is calculating, and how you are going to do the calculation. put the units in at the end of each step, not in your working.

Sami pays $25 \times 0{\cdot}84 = 50 \times 0{\cdot}42 = 100 \times 0{\cdot}21 = £21$
He sells 21 of them at £1·40 each, so gets $21 \times 1{\cdot}4 = 20 \times 1{\cdot}4 + 1 \times 1{\cdot}4 = 28 + 1{\cdot}4 = £29{\cdot}40$
So he makes £8·40 profit, on his original payment of £21, so

$$\text{percentage profit} = \frac{8\cdot 40}{21} \times 100 = \frac{840}{21} = 40\%.$$

2. A *Finest* cream dessert is sold in tubs of 450ml which contain 125ml of cream. A *Superb* cream dessert is sold in tubs of 375ml which contain 105ml of cream.

Which cream dessert contains the greater proportion of cream?

Solution

$$\textit{Finest} \text{ proportion} = \frac{125}{450} = \frac{25}{90} = \frac{5}{18}$$
$$\textit{Superb} \text{ proportion} = \frac{105}{375} = \frac{21}{75} = \frac{7}{25}$$

Comparing fractions is best done by putting them over a common denominator.

Lowest common multiple of 18 and 25 is $18 \times 25 = 450$.

Proportions are: *Finest* $\frac{125}{450}$; *Superb* $\frac{7}{25} = \frac{18 \times 7}{18 \times 25} = \frac{126}{450}$

So the *Superb* dessert contains a very slightly higher proportion.

Don't forget to state your conclusion: don't leave the readers of your work to look at your calculations and decide for themselves, even if you think it's obvious.

3. Solve the equations:

a) $\frac{1}{6}x - \frac{1}{4}(x-5) = 1$

b) $\frac{x}{x-1} = \frac{x+1}{x-2}$

Solution

The easiest way to solve equations involving fractions is usually to multiply both sides of the equation by the simplest common multiple of the denominators.

a Lowest common multiple of 4 and 6 is 12, so multiply both sides of the equation by 12.

$$12 \times \tfrac{1}{6}x - 12 \times \tfrac{1}{4}(x-5) = 12 \times 1$$

so $2x - 3(x-5) = 12$

so $2x - 3x + 15 = 12$

so $x = 3$

b The simplest common multiple of $(x-1)$ and $(x-2)$ is $(x-1)(x-2)$, so multiply both sides of the equation by $(x-1)(x-2)$.

$$(x-1)(x-2)\frac{x}{(x-1)} = (x-1)(x-2)\frac{(x+1)}{(x-2)}$$

It is often helpful to put extra brackets in, as has been done here, to make sure you remember to treat the numerator or denominator of an algebraic fraction as a single factor, as you should.

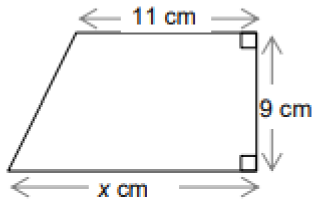
so $(x-2)x = (x-1)(x+1)$

so $x^2 - 2x = x^2 - 1$

so $x = \frac{1}{2}$

In both parts of this question, it would be sensible to check your answer by putting your value of x back into the original equation.

4. The area of this trapezium is $117cm^2$. What is the length x cm of its base?



Solution

You should know the formula for the area of a trapezium: (average of parallel sides) × height.

If you knew x you could work out the area of the trapezium by calculating

$$\frac{(x+11)}{2}\times 9$$

But you know this must come to 117, so you have the equation

$$\frac{(x+11)}{2}\times 9 = 117$$

so $(x+11) = \frac{234}{9} = 26$

so $x = 15$.

So the length of the base is 15 cm.

5. The five-digit numbers 91723 and 85604 use all ten digits between them. The difference between these numbers is 91723 – 85604 = 6119.

Find two five-digit numbers which use all ten digits between them and which have the *smallest possible* difference.

Solution

To make the difference as small as possible, you want the two numbers to have first digits which differ by 1: this makes the numbers 10000 apart, but you can reduce this difference by making the number with the larger first digit have the least possible added to it from the remaining four digits, and the number with the smaller first digit have the most possible added to it from the remaining four digits.

This means that you want 0123 be the last four digits of the number with the larger first digit, and 9876 to be the last four digits of the number with the smaller first digit. this leaves 4 and 5 as the first digits, so the numbers are

50123

and 49876,

which differ by 247.

6. Find three different whole numbers A, B and C, so that

- B is the average of A and C
- A^2 is the average of B^2 and C^2

Note that not all of the numbers can be positive

Solution

This question is intended to be done by juggling with the numbers.
The simplest solution is probably $A = -5$, $B = 1$ and $C = 7$.

You could also tackle the question algebraically, using the equations

$$B = \frac{A+C}{2} \text{ and } A^2 = \frac{B^2+C^2}{2}.$$

The first equation gives $A = 2B - C$ which you can substitute into the second:

$$B^2 + C^2 = 2(2B-C)^2.$$

This simplifies to

$$7B^2 - 8BC + C^2 = 0$$

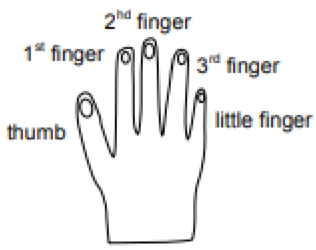
or, factorising,

$$(7B-C)(B-C) = 0$$

So $C = 7B$ (since C and B are different).

Be reassured: in the test, you would not be expected to solve this problem algebraically.

7. A boy counts on his fingers, backward and forwards across his right hand as follows: thumb, 1st finger, 2nd finger, 3rd finger, little finger, 3rd finger, 2nd finger, 1st finger, thumb, 1st finger, and so on.



If he starts counting at one on his thumb, which finger will he be on when he reaches two thousand and thirteen?

Explain clearly how you decided.

Solution

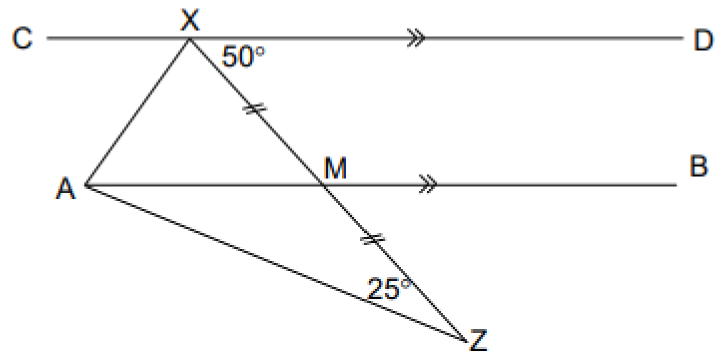
"Explain clearly", in this question, means you cannot just show a calculation, but you need to say in words why that calculation gives the answer you want.

The boy counts his thumb on 1, 9, 17, and so on: that is on numbers that are one more than a multiple of eight.
The last multiple of eight before 2013 is 2008, so the boy will count on his thumb on 2009.
Therefore he will count on his
1st finger on 2010
2nd finger on 2011
3rd finger on 2012
and little finger on 2013.

An answer without a suitable explanation might look like this.
$2013 = 8 \times 251 + 5$ so little finger.

8. In the diagram, line CD is parallel to line AB, and M is the midpoint of line XZ.

Angle DXM = 50° and angle MZA = 25°



Find angles

i) MAZ

ii) CXA

showing and explaining each step in your working.

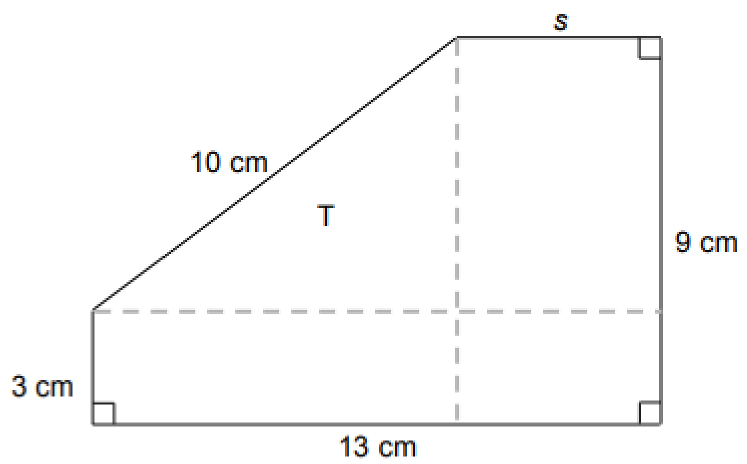
Solution

In geometry problems, when you are asked to explain your working, write each statement on a new line, with a reason why the statement is true (usually in brackets afterwards).

Angle BMZ = 50° (corresponding with angle DXM on parallel lines CD and AB).
So angle AMZ = 180 – 50 = 130° (angles on a straight line add to 180°).
So angle MAZ = 180 – 130 – 25 = 25° (angles in a triangle add to 180°).
This means that triangle ZMA is isosceles (base angles MAZ and MZA are equal).
So AM = MZ (sides opposite base angles of an isosceles triangle).
So AM = XM (it is given that M is the midpoint of XZ, so MZ = XM).
So triangle XMA is isosceles (two equal sides).
So angle MAX = angle MXA (base angles of isosceles triangle MAX).
But angle XMA = 50° (alternate with angle DXM on parallel lines CD and AB).
So angle MAX = $\frac{1}{2}(180-50)=65°$ (angles in a triangle add to 180°).

9. The diagram shows an irregular pentagon. The lengths of four of the sides are shown in the diagram. Three of the angles in the pentagon are right angles, as shown.

Find the length of the side marked s.



Solution

The dashed lines have been added to the diagram. Right angled triangle T has been marked.

You can see that the height of the right angled triangle is 9 – 3 = 6 cm, but its hypotenuse is 10 cm, so by Pythagoras's Theorem its base is

$$\sqrt{10^2 - 6^2} = \sqrt{64} = 8 \text{ cm.}$$

This means that s = 13 – 8 = 5 cm.

10. Five bananas and two kiwi fruit cost £2.30, and four bananas and three kiwi fruit cost £2.47.

How much would it cost to buy two bananas and one kiwi fruit?

Solution

In algebra problems, it is a good idea to state clearly at the start of the question what x stands for.

Let x be the cost in pence of a banana, and y the cost of a kiwi fruit in pence.
Then, working in pence throughout,

$$5x + 2y = 230$$
$$4x + 3y = 247$$

Multiplying the first equation by 3, and the second by 2, gives

$$15x + 6y = 690$$
$$8x + 6y = 494$$

and subtracting the equations then gives

$$7x = 196$$

so $x = 28$ and

$$5 \times 28 + 2y = 230$$

so $y = 45$.
A banana costs 28p and a kiwi fruit 45p.

11. In September, there were three times as many boys as girls in a class.

In October, two new girls joined the class and one of the boys was expelled. Now there were only twice as many boys as girls in the class.

How many boys and girls were there in the class in September?

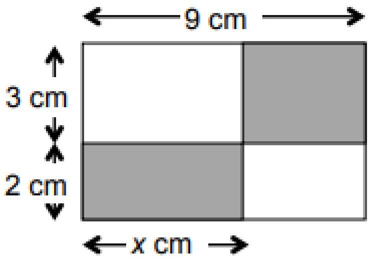
Solution

Let x be the number of girls in the class at the start of the year.
Then there are $3x$ boys in the class at the start of the year.
After Christmas, there were $x + 2$ girls and $3x - 1$ boys in the class.
So

$$3x - 1 = 2(x + 2)$$

so $x = 5$.
There were 5 girls and 15 boys in the class initially.

12. The two shaded rectangles have equal area. What is the total shaded area?



Solution

Let x cm be the width of the lower shaded rectangle, as shown on the diagram.
Then the width of the upper shaded rectangle is $(9 - x)$ cm
So, because the areas are equal,

$$2x = 3(9 - x).$$

so $x = \frac{27}{5}$ (= 5·4 , if you like).

So the shaded area = $2 \times \left(2 \times \frac{27}{5}\right) = \frac{108}{5}$ cm^2 (= 21·8 cm^2, if you like).

13. To **double** a number means to multiply it by two.

In this question, we define 'to **twiddle**' a number means adding four to it, and 'to **flip**' a number will mean to subtract it from 8 (so flipping 3 gives 5, flipping -1 gives 9 etc.)

a) If you twiddle a number and then twiddle the answer, the overall effect is to add eight to the number.

What would the overall effect of the two operations be if you:

i) flip a number and then flip the answer?

ii) twiddle a number and then flip the answer?

iii) flip a number and then twiddle the answer?

b) Show that if you twiddle a number, flip the answer and then twiddle the answer to that, this has the same overall effect as just flipping once.

c) Find a sequence of three operations, each a twiddle or a flip, that has the overall effect of just changing the sign of the number (e.g. 3 becomes -3, or -7 becomes 7)

Solution

a **i** The number stays the same.
ii The overall effect is the same as subtracting the number from 4.
iii The overall effect is the same as subtracting the number from 12.

b If the starting number is x, twiddling it gives $x + 4$, flipping this answer gives

$$8 - (x + 4) = 4 - x$$

and twiddling this answer gives

$$(4 - x) + 4 = 8 - x,$$

which is the same outcome as just flipping x once.

c twiddle, followed by twiddle, followed by flip has the outcome

$$8 - ((x + 4) + 4) = - x$$

when applied to x.